43. $x = 3t - 2\sin t$
$y = 3 - 2\cos t$

arc length.

$$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \text{arc length for parametric eqns}$$

$$\int_0^{2\pi} \sqrt{(3-2\cos t)^2 + (2\sin t)^2}\, dt$$

$\geq$.

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10.2 vectors, vector valued functions

magnitude, direction
(length)

$\hat{v}$, $\vec{v}$

components

$\hat{v} = \ell \angle \theta$ polar

$\hat{v} = \hat{x} + \hat{y}$ rectangular

$\hat{v} = \langle x\cos\theta, y\sin\theta \rangle$

x & y components

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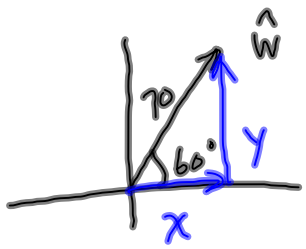

$\hat{w} = 70\angle 60^\circ$

$= \langle 70\cos 60^\circ, 70\sin 60^\circ \rangle$

$x \qquad y$

$70\cos 60^\circ = \frac{x}{70}$

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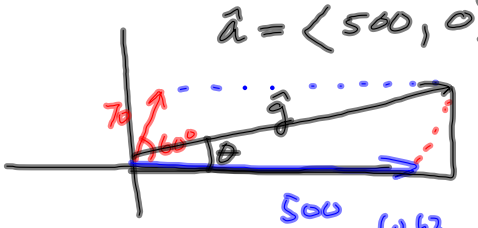
airplane $\hat{a} = 500$ mph, due east

air speed

wind $\hat{w} = 70$ mph, 60° N of E

$\hat{a} = \langle 500, 0 \rangle$ $\hat{w} = \langle 70\cos 60^\circ, 70\sin 60^\circ \rangle$



$\hat{g} = \langle 535, 35\sqrt{3} \rangle$

$\theta = \tan^{-1}\left(\frac{60.62}{535}\right)$

$\theta = 6.5^\circ$ N of E

speed $= |\hat{g}| = \sqrt{535^2 + 60.62^2} = 538$

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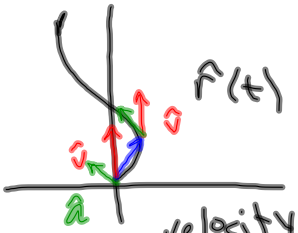
vector-valued functions

position vector = $\hat{r}(t) = \langle \sin t, \frac{t^2}{2} \rangle$

use parametric equations $X1 = \sin t$

$Y1 = \frac{t^2}{2}$



$t = \frac{\pi}{2} \approx 1.57$

$\hat{r}(\frac{\pi}{2}) = \langle 1, 1.23 \rangle$

velocity: $\hat{v}(t) = \langle \cos t, t \rangle$

$\hat{v}(\frac{\pi}{2}) = \langle 0, \frac{\pi}{2} \rangle$

$\hat{a} = \langle -\sin(t), 1 \rangle$ $\hat{a}(\frac{\pi}{2}) = \langle -1, 1 \rangle$

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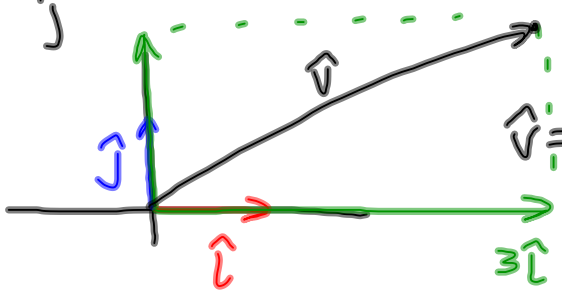
Unit vectors (length of 1 unit)

$\hat{i}, \hat{j}$



$\hat{v} = 3\hat{i} + 2\hat{j}$

or

$\hat{v} = \langle 3, 2 \rangle$

or

$\hat{v} = (3, 2)$

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Ex 8. $\vec{v} = \langle t - 3\pi\cos(\pi t),\ 2t - \pi\sin(\pi t) \rangle$ ($\frac{dx}{dt}$, $\frac{dy}{dt}$)

$\vec{r}(0) = \langle 1, 5 \rangle$

a) find $\vec{r}(4)$

1st: find $\vec{r}(t)$

$$\vec{r}(t) = \left\langle \frac{t^2}{2} - 3\sin(\pi t) + c_1,\ t^2 + \cos(\pi t) + c_2 \right\rangle$$

$1 = 0 - 0 + c_1$ $\qquad$ $5 = 0 + 1 + c_2$

$1 = c_1$ $\qquad$ $4 = c_2$

$$\vec{r}(t) = \left\langle \frac{t^2}{2} - 3\sin(\pi t) + 1,\ t^2 + \cos(\pi t) + 4 \right\rangle$$

$$\vec{r}(4) = \left\langle \frac{4^2}{2} - 3\sin(4\pi) + 1,\ 4^2 + \cos(4\pi) + 4 \right\rangle$$

$$= \langle 9, 21 \rangle = 9\hat{i} + 21\hat{j}$$

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distance traveled = arc length

$$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

components of velocity

$$\int_0^4 \sqrt{\left(t - 3\pi\cos(\pi t)\right)^2 + \left(2t - \pi\sin(\pi t)\right)^2}\, dt$$

$\int$ speed = distance

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$$\text{speed} = |\hat{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

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